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Isospin and $Z^{1/3}$-dependence of the nuclear charge radii

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Abstract. Based on the systematic investigation of the data available for nuclei with $A \geq 40$, a $Z^{1/3}$-dependence for the nuclear charge radii is shown to be superior to the generally accepted $A^{1/3}$ law. A delicate scattering of data around $R_c/Z^{1/3}$ is inferred as owing to the isospin effect and a linear dependence of $R_c/Z^{1/3}$ on N/Z (or $(N-Z)/2$) is found. This inference is well supported by the microscopic Relativistic Continuum Hartree-Bogoliubov (RCHB) calculation conducted for the proton magic Ca, Ni, Zr, Sn and Pb isotopes including the exotic nuclei close to the neutron drip line. With the linear isospin dependence provided by the data and RCHB theory, a new isospin-dependent $Z^{1/3}$ formula for the nuclear charge radii is proposed.

PACS. 21.10.-k Properties of nuclei; nuclear energy levels – 21.10.Ft Charge distribution – 21.10.Dr Binding energies and masses – 21.60.-n Nuclear-structure models and methods

The nuclear radius is one of the most fundamental bulk properties of an atomic nucleus [1,2]. Among all the size quantities describing the nucleus, nuclear charge radii have been investigated by various techniques and methods experimentally [3–9], including the muonic atom spectroscopy [3], the isotope shift of optical and K X-ray spectroscopy [4–8] and high-energy elastic electron scattering [9], etc. Recently more and more nuclei far from the β-stability line have become accessible experimentally thanks to the development of radioactive ion beam facilities [10,11]. The nuclear size connected with exotic phenomena such as skin and halo has become a hot topic. The understanding of its property has importance not only in nuclear physics, but also in other fields such as astrophysics and atomic physics, etc. With its accuracy, the study of the nuclear charge radii is very important to understand not only the proton distribution inside the nucleus but also the halo and skin. Particularly, if one can get a simple and reliable formula for nuclear charge radii, it will be very useful to extract the de-coupling of proton and neutron in the exotic nuclei and provide information for the effective nucleon-nucleon interaction widely used in all the nuclear models. In this paper the available charge radii data for $A \geq 40$ will be examined and their global behavior will be studied. Instead of the widely accepted $A^{1/3}$ law, a new $Z^{1/3}$ formula with isospin effect will be proposed.

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Based on the consideration of the nuclear saturation property, the nuclear charge radius R_c is usually described by the $A^{1/3}$ law [1,2]:

$$R_c = r_A A^{1/3}, \tag{1}$$

where A is the mass number and $R_c = \sqrt{\frac{5}{3}}\,\langle r^2\rangle^{1/2}$, with $\langle r^2\rangle^{1/2}$ the root-mean square (rms) charge radius. For very light nuclei, because of their small A and large fluctuation in charge distribution due to the shell effect with short period, it seems that the charge distribution radius as a bulk property has little meaning. A detailed analysis of charge radii data for $A \geq 40$ shows that r_A is by no means a constant, but systematically decreases with A; *i.e.*, $r_A \approx 1.31$ fm for light nuclei ($A \sim 40$) and $r_A \approx 1.20$ fm for very heavy nuclei (see upper left panel in fig. 1). This fact implies that some physics is missing in eq. (1).

Definite evidences of the violation of $A^{1/3}$ law are also found in the measurements of isotope shift in root mean square charge radii [12,13]. In particular, $\delta\langle r^2\rangle_{A+2,A}$ values (associated with an addition of two neutrons) are often found to be considerably smaller compared to what is expected from the $A^{1/3}$ law ($\delta\langle r^2\rangle_{A+2,A} = \frac{4}{3A}\langle r^2\rangle_A$). A typical example is that the observed charge radii of the calcium isotopes $^{40\text{-}50}$Ca remain almost the same (except a very little change induced by deformation or shell effect), though the mass number A has changed significantly. In contrast, there is also evidence that the observed $\delta\langle r^2\rangle_{A+2,A}$ values (associated with the addition of

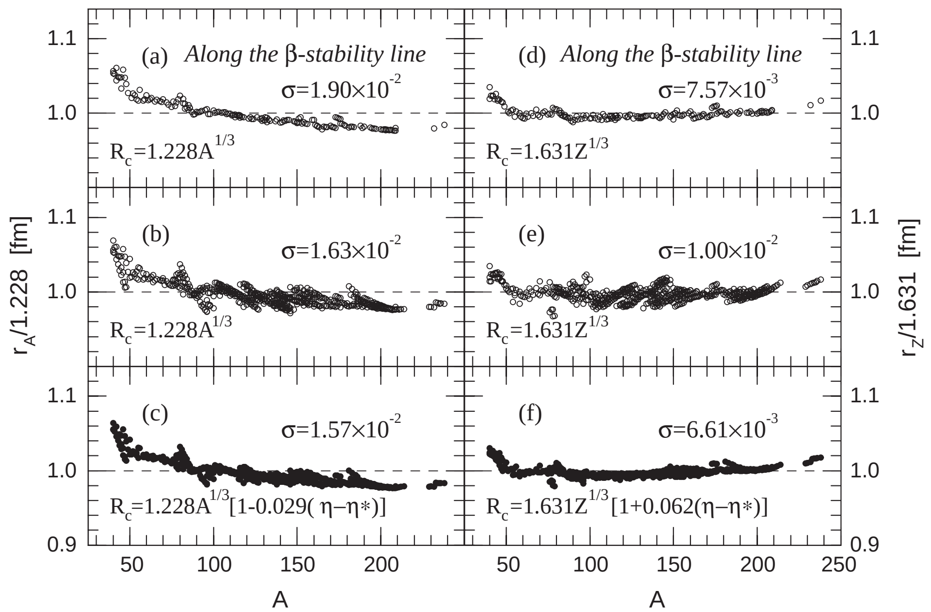

Fig. 1. The nuclear charge radius data for r_A in the $A^{1/3}$ and r_Z in the $Z^{1/3}$ law with and without isospin dependence, for the details see the text.

two protons) are often greater than what is expected from the $A^{1/3}$ law (*e.g.*, $\delta\langle r^2\rangle$ for ^{46}Ti-^{44}Ca, ^{50}Ti-^{48}Ca, etc.).

Along the β-stability line, the ratio Z/A gradually decreases with A, *i.e.*, for light nuclei $Z/A \approx 1/2$, and for the heaviest β-stable nucleus $^{238}_{92}$U, $(Z/A)^{1/3} \approx 0.7285$. It turns out that $(1/2)^{1/3}/0.7285 \approx 1.09$, which is very close to the r_A ratio 1.30/1.20 shown in the upper left panel of fig. 1. A naive point of view is that the charge radius of a nucleus may be more directly related to its charge number Z, rather than its mass number A. Therefore, compared to the $A^{1/3}$ law, a $Z^{1/3}$-dependence for nuclear charge radii may be more reasonable

$$R_c = r_Z Z^{1/3}\,, \tag{2}$$

as noted in ref. [12]. An analysis of the very limited data of charge radii then available showed that r_Z remains almost a constant, *i.e.*, $r_Z = 1.65(2)$ fm for $A \geq 40$. A similar $Z^{1/3}$-dependent formula for nuclear charge radii was also proposed in ref. [14]. With the physics for such a simple $Z^{1/3}$ law open, we note here that the $Z^{1/3}$ law of R_c includes an isospin dependence compared to the $A^{1/3}$ law as follows:

$$\begin{aligned}R_c = r_Z Z^{1/3} &= r_Z\left(\frac{A}{2}-\frac{N-Z}{2}\right)^{1/3}\\ &= \frac{r_Z}{2^{1/3}}A^{1/3}\left(1-\frac{N-Z}{A}\right)^{1/3}\\ &\approx \frac{r_Z}{2^{1/3}}A^{1/3}\left(1-\frac{1}{3}\frac{N-Z}{A}\right).\end{aligned} \tag{3}$$

One can get $r_A = r_Z/2^{1/3} \approx 1.31$ fm and 0.33 for the coefficient of the isospin term. Equation (3) is exactly the isospin-dependent $A^{1/3}$ formula, $R_c = 1.25A^{1/3}\left(1-0.2\frac{N-Z}{A}\right)$, proposed in ref. [13]. Even the coefficients in eq. (3) are very close to those obtained in ref. [13] by fitting the data. Obviously eq. (2) has the merits that only one parameter is needed.

The $Z^{1/3}$-dependence of nuclear charge radii was also used to modify the Coulomb energy term in the semi-empirical nuclear-mass formula [15], and it was found that the agreement between the calculated and experimental results was improved. Moreover, the $A^{-1/3}$ law for the nuclear giant (monopole, dipole and quadrupole) resonance energy ($\propto 1/R$) also could be improved, if the $A^{-1/3}$-dependence is replaced by a $Z^{-1/3}$-dependence [16].

In the past two decades, a vast amount of new experimental information on the electromagnetic structure of nuclear ground states of many nuclei has become available [3–9], and accuracy has been improved. In particular the muon factories at Los Alamos (LAMPF) and at Villigen (PSI, formerly SIN) started their operation in 1974. Almost all stable nuclei have been measured by the muonic X-ray transition technique and the corresponding charge radii have been rather accurately deduced (the experimental relative error is about 10^{-3}). Moreover, modern techniques for optical isotope shift measurements have made it possible to reach even short-lived (down to 1 s) unstable isotopes [3]. Therefore, it is worthwhile to re-examine the fundamental property of nuclei and to investigate whether the vast amount of improved experimental results follow the $Z^{1/3}$-dependence. The values of measured $\langle r^2\rangle^{1/2}$ for 536 nuclei with $A \geq 40$ compiled in refs. [3–9] are analyzed in fig. 1 by using the $A^{1/3}$- and $Z^{1/3}$-dependence, respectively. The dependence of charge radii on the quadrupole deformation β has been taken into account for the rare-earth deformed nuclei by [1]

$$r_A = r_{Ad}\left(1+\frac{5}{8\pi}\beta^2\right), \qquad r_Z = r_{Zd}\left(1+\frac{5}{8\pi}\beta^2\right), \tag{4}$$

and the values of β are taken from refs. [9,17]. Apparently, for spherical nuclei ($\beta = 0$), $r_A = r_{Ad}$, $r_Z = r_{Zd}$.

In the upper left and right panels of fig. 1, the charge radii for the most stable 159 nuclei with $A \geq 40$ along the β-stability line have been analyzed by using the $A^{1/3}$- and $Z^{1/3}$-dependence. In the middle left and right panels, the same has been done for the measured $\langle r^2 \rangle^{1/2}$ for 536 nuclei with $A \geq 40$. Two significant features can be observed: A) On the one hand, the agreement between the data and the calculated results using the $Z^{1/3}$-dependence is much better than that using the $A^{1/3}$ law, *i.e.*, while there exists a global regular decrease of r_{Ad} with A, r_{Zd} nearly remains constant ($r_{Zd} = 1.631(11)$ fm). The relative rms deviations σ for the $Z^{1/3}$-dependence ($\sigma = 7.57 \times 10^{-3}$ for stable nuclei and 1.00×10^{-2} for 536 nuclei) are much less than those for the $A^{1/3}$ law ($\sigma = 1.90 \times 10^{-2}$ for stable nuclei and 1.63×10^{-2} for 536 nuclei). B) On the other hand, though the rms deviation for the $Z^{1/3}$-dependence is significantly reduced, an isospin-induced scattering of the data in the middle panels in fig. 1 can be also observed compared with that in the top panels. In fact, r_{Zd} generally increases with N for most isotopic chains, *e.g.* for ${}^{90\text{-}96}_{40}$Zr, ${}^{92\text{-}100}_{42}$Mo, ${}^{96\text{-}104}_{44}$Ru, ${}^{102\text{-}110}_{46}$Pd, ${}^{106\text{-}116}_{48}$Cd, ${}^{112\text{-}124}_{50}$Sn, ${}^{122\text{-}130}_{52}$Te, ${}^{124\text{-}136}_{54}$Xe, ${}^{142\text{-}148}_{60}$Nd, ${}^{144\text{-}154}_{62}$Sm, ${}^{154\text{-}160}_{64}$Gd, etc, (except for only a few lighter isotopic chains, *e.g.*, ${}^{78\text{-}86}_{36}$Kr, ${}^{84\text{-}88}_{38}$Sr, and a small anomalous decrease of r_{Zd} with N due to the shell closure at $N = 50$. Therefore, it seems necessary to investigate an isospin-dependent correction for the scattering of r_{Zd}. In refs. [13, 18–20], the isospin effects has been considered based on the $A^{1/3}$ law. However, considering the fact that the $Z^{1/3}$-dependence can describe the nuclear charge radii much better than the $A^{1/3}$ law, we take the $Z^{1/3}$-dependence as a more reasonable starting point for describing the isospin dependence of nuclear charge radii.

To make the isospin-dependent $Z^{1/3}$ formula to be developed for nuclear charge radii be based on more strong foundation and be also valid for exotic nuclei, the charge radii of nuclei far from the β-stability line up to drip lines are needed. However, such data is not easily available, an alternative is to require that our new isospin-dependent $Z^{1/3}$ formula should assort with a reliable and microscopic nuclear model.

The fully self-consistent and microscopic relativistic continuum Hartree-Bogoliubov (RCHB) theory, which is an extension of the relativistic mean field (RMF) [21–23] and the Bogoliubov transformation in the coordinate representation [24], is a good candidate for the present purpose. The RCHB theory has been used satisfactorily to describe lots of the ground-state properties for spherical nuclei and to understand the pseudo-spin symmetry in finite nuclei [25,26]. A remarkable success of the RCHB theory is the self-consistent reproduction of the halo in ^{11}Li [27] and the prediction of the exotic phenomenon — giant halo [28]. In combination with the Glauber model, the RCHB theory successfully reproduces the interaction cross-section in Na isotopes [29] and the charge changing cross-section of C, N, O, F isotopes (ranging from the β-stability line to the neutron drip line) on the target of ^{12}C at 930 MeV/u [30]. These successes encourage us to apply the RCHB theory for the description of charge radii of nuclei both close to and far from the β-stability line and check its validity for the data available and provide information for nuclei far away from the stability line.

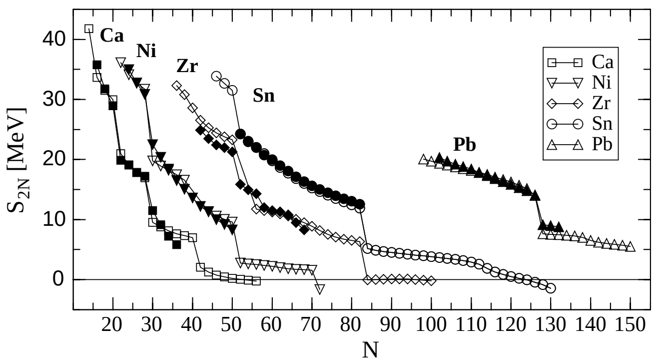

Fig. 2. Two-neutron separation energies S_{2n} of even Ca, Ni, Zr, Sn, Pb isotopes as a function of N, including the data (solid symbols) from ref. [32] and the RCHB calculation with a δ-force (open symbols).

The detailed formalism and numerical techniques of the RCHB theory can be found in ref. [24] and references therein. In the present calculations, we follow the procedures in refs. [24,28,29] and solve the RCHB equations in a box with the size $R = 20$ fm and a step size of 0.1 fm. As in refs. [24–26,28–30], the parameter set NL-SH [31] is used, which aims at describing both the stable and exotic nuclei. Other parameter sets produce similar results and do not change the following conclusions. The density dependent δ-force in the pairing channel with $\rho_0 = 0.152$ fm^{-3} is used and its strength V_0 is fixed by the Gogny force as in ref. [24]. The contribution from continua is restricted within a cut-off energy $E_{\text{cut}} \sim 120$ MeV.

As typical examples, we studied the even-even Ca, Ni, Zr, Sn and Pb isotopes ranging from the β-stability line to neutron drip line. The two-neutron separation energies S_{2n} is one of the essential quantities to test a nuclear model. In fig. 2, the calculated S_{2n} (open symbols) of the even-even Ca, Ni, Zr, Sn and Pb isotopes by the RCHB theory are compared with the data available (solid symbols) [32], where a satisfactory agreement is seen. Particularly the deviation between the calculated binding energies with the data available is within 1%. In the present calculation, the neutron drip line nuclei are predicted at ^{72}Ca, ^{98}Ni, ^{140}Zr and ^{176}Sn, respectively. In the S_{2n} *versus* N curve for each isotopic chain, there are some kinks due to the neutron shell or subshell closure. For example, the closed shells at $N = 20$, 28 and subshell at $N = 40$ correspond to kinks in the S_{2n} *versus* N curve for Ca isotopes at ^{40}Ca, ^{48}Ca and ^{60}Ca, respectively. Note that the kink at $N = 20$ for ^{40}Ca may be also due to the Wigner term for $N = Z = 20$. However, there are no kinks at ^{70}Ca and ^{176}Sn, which indicate the disappearance of the magic numbers $N = 50$ and 126 for these nuclei in RCHB.

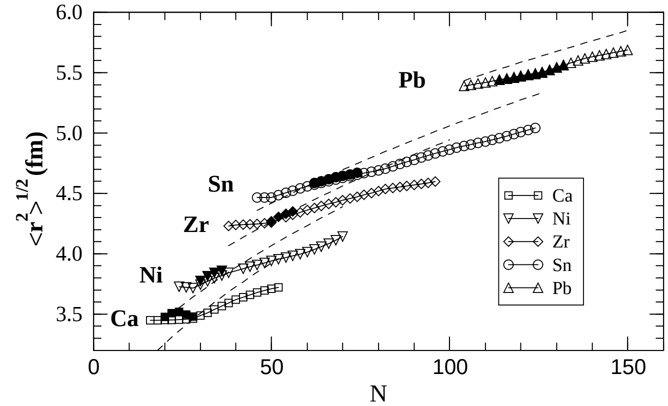

Fig. 3. The rms charge radii *versus* the neutron number N for even-even Ca, Ni, Zr, Sn, Pb isotopes. The RCHB calculation with δ-force is represented by open symbols, while the corresponding data is denoted by solid symbols. The dashed lines represent the predictions by the $A^{1/3}$ law with $r_{Ad} = 1.228$ fm.

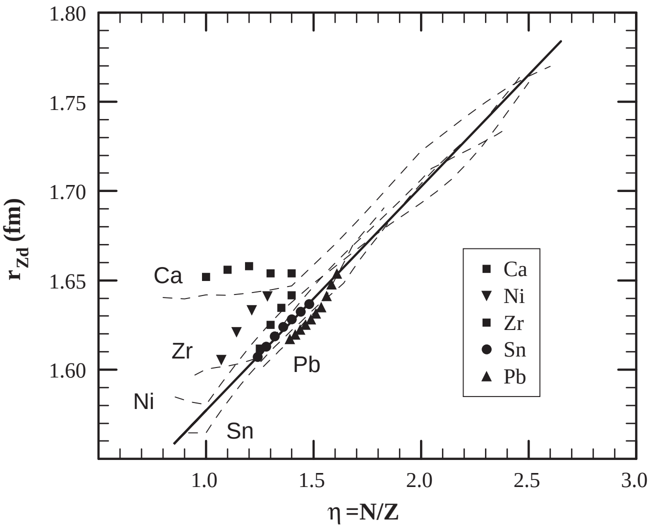

Fig. 4. The experimental (solid symbols) and RCHB-predicted (dashed lines) coefficient $r_{Zd} = R_c/Z^{1/3}$ for the nuclear charge radii as a function of the isospin quantity $\eta = N/Z$ in even-even Ca, Ni, Zr, Sn and Pb isotopes. The asymptotic behavior is drawn as a solid line.

The rms charge radii $\langle r^2\rangle^{1/2}$ obtained from the RCHB theory (open symbols) and the data available (solid symbols) for the even-even Ca, Ni, Zr, Sn and Pb isotopes are given in fig. 3. As it could be seen, the RCHB calculations reproduce the data very well (within 1.5%). For a given isotopic chain, an approximate linear N-dependence of the calculated rms charge radii $\langle r^2\rangle^{1/2}$ is clearly seen in fig. 3, which shows that the variation of $\langle r^2\rangle^{1/2}$ for a given isotopic chain deviates from both the simple $Z^{1/3}$-dependence and the simple $A^{1/3}$ law (denoted by dashed lines in fig. 3). Therefore, a strong isospin dependence of nuclear charge radii is necessary for nuclei with extreme N/Z ratio.

In fig. 4, the experimental and RCHB predicted $r_{Zd} = R_c/Z^{1/3}$ for the proton magic isotopes are presented as a function of $\eta = N/Z$. It is clearly seen that the coefficient r_{Zd} increases linearly with η (except for some deviations due to deformation or shell effects) and the slopes are nearly the same for these isotopic chains. The linear η (or isospin $T_Z = (N-Z)/2$) dependence of r_{Zd} for an isotopic chain may be understood as the effect of the first-order perturbation correction of the nuclear wave function due to an isospin T_Z-dependent interaction [33]. Based on the analysis of data in the middle and upper panels of fig. 1 and RCHB prediction in figs. 3 and 4, we propose the following isospin-dependent $Z^{1/3}$ formula for nuclear charge radii:

$$R_c = r_Z\, Z^{1/3}\, [1 + b(\eta - \eta^*)]\,, \qquad \eta = N/Z, \tag{5}$$

where η^* is $\eta = N/Z$ for the nuclei along the β-stability line which can be directly extracted from the nuclear-mass formula [1], $r_Z = r_{Zd}(1 + \frac{5}{8\pi}\beta^2)$, $r_{Zd} = 1.631(11)$ fm as obtained in the upper right panel of fig. 1, and $b = 0.062(9)$ obtained from the least-square fitting.

The analysis of the available data using eq. (5) with r_{Zd} and b thus obtained is displayed in the lower right panel of fig. 1. It is found that the data are reproduced better by eq. (5) than by eq. (2) (the rms deviation is reduced by about 40%). Description of the data with an isospin-dependent $A^{1/3}$ formula, $R_c = 1.228\, A^{1/3}\, [1 + 0.029(\eta - \eta^*)]$ is also shown in the lower left panel of fig. 1 for comparison. The parameter for the isospin term is also obtained by least-square fitting of the data. It can be seen that this formula describes the data less successfully than eq. (5). It is expected that the modified $Z^{1/3}$ formula (eq. (5)) will become more useful with more and more data obtained for the nuclei far from the β-stability line.

In refs. [13,18], a modified $A^{1/3}$ formula with additional terms $(N-Z)/A$ and $1/A$ made some successes in describing the nuclear charge radius. Dobaczewski *et al.* [19] extended this formula with more terms, *e.g.*, A^{-2}, to account for the isospin dependence of charge radii in light nuclei. Based on the idea that $R_p Z^{-1/3}$ is approximately a constant (R_p is the rms radius of the proton density distribution), Gambhir and Patil [14] proposed a formula for the nuclear charge radius: $R_c = ((r_p Z^{1/3})^2 - 0.64)^{1/2}$. A comparison between these formulae and the presently proposed one, eq. (5), is given in table 1. All the coefficients in these formulae are obtained by least-square fitting to the available charge radii data of $A \geq 40$ nuclei. Corresponding relative rms deviations of each formula with data are listed as well.

From table 1 one can observe that: i) the $Z^{1/3}$-dependence is a better approximation to available R_c data than the $A^{1/3}$ one; ii) the simple $Z^{1/3}$-dependent formula, eq. (2) gives nearly the same result as the formula suggested by Gambhir and Patil; iii) the presently proposed isospin dependent $Z^{1/3}$ formula, eq. (5), is the best among the two-parameter isospin-dependent formulae and even gives comparable results as the three-parameter one. We note that better agreement of the isospin-dependence $A^{1/3}$ formulae of charge radii is achieved by fitting both r_A and b (or c) simultaneously. While in eq. (5), we use the

Table 1. Comparison between the available formulae for the nuclear charge radius R_c. The parameters in the first, third, fourth and fifth formula are obtained by least-square fitting to the data with $A \geq 40$, while those for the second and sixth formulas are the same as in fig. 1 .

Formula	Parameters	σ	References
$R_c = r_A A^{1/3}$	$r_A = 1.223$ fm	1.55×10^{-2}	[1,2]
$R_c = r_Z Z^{1/3}$	$r_Z = 1.631$ fm	1.00×10^{-2}	[12]
$R_c = \sqrt{5/3}\left((r_p Z^{1/3})^2 + 0.64\right)^{1/2}$	$r_p = 1.242$ fm	9.09×10^{-3}	[14]
$R_c = r_A\left(1 - b\frac{N-Z}{A}\right)A^{1/3}$	$r_A = 1.269$ fm; $b = -0.252$	8.27×10^{-3}	[13]
$R_c = r_A\left(1 - b\frac{N-Z}{A} + c\frac{1}{A}\right)A^{1/3}$	$r_A = 1.235$ fm; $b = -0.177$; $c = 1.960$	4.21×10^{-3}	[18]
$R_c = r_Z\left(1 + b\frac{N-N^*}{Z}\right)Z^{1/3}$	$r_Z = 1.631$ fm; $b = 0.062$	6.61×10^{-3}	Present

same r_Z as that in eq. (2), which are derived only from least-square fitting to the most stable nuclei. Thus, the second term in eq. (5) mainly corresponds to the isospin dependence of charge radii for the nuclei away from the β-stability line.

In summary, we have systematically investigated the nuclear charge radii with $A \geq 40$. It is clearly seen that the $Z^{1/3}$-dependence is superior to the $A^{1/3}$ law. A delicate scattering of data around $R_c/Z^{1/3} = 1.631$ is inferred as due to the isospin effects and a linear dependence of $R_c/Z^{1/3}$ on N/Z (or $(N-Z)/2$) is found. This inference is well supported by the microscopic RCHB calculation conducted for the proton magic Ca, Ni, Zr, Sn and Pb isotopes including the exotic nuclei close to the neutron drip line, which reproduce S_{2n} and nuclear charge radii data available well. With the linear dependence of the coefficient r_{Zd} on N/Z (or $(N-Z)/2$) read from the data and RCHB theory, a new isospin-dependent $Z^{1/3}$ formula for nuclear charge radii is proposed, which improves the description of the data available for nuclei near the β-stability line and may be more suitable for new data on nuclei far from the β-stability line.

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References

1. A. Bohr, B.R. Mottelson, *Nuclear Structure*, Vol. **1** (W.A. Benjamin Inc., New York, Amesterdam, 1969).
2. P. Ring, P. Shuck, *The Nuclear Many-Body Problem* (Springer-Verlag, Berlin, 1980).
3. G. Fricke *et al.*, At. Data Nucl. Data Tables **60**, 177 (1995) and references therein.
4. P. Aufmuth, K. Heilig, A. Steudel, At. Data Nucl. Data Tables **37**, 455 (1987).
5. E.W. Otten, *Treatise on Heavy-Ion Science*, edited by D.A. Bromley, Vol. **8** (Plenum, New York, 1988) p. 515.
6. J. Billowes, P. Campbell, J. Phys. G: Nucl. Part. Phys. **21**, 707 (1995) and references therein.
7. H.A. Schuessler, A. Alousi, R.M. Evans *et al.*, Phys. Rev. Lett. **65**, 1332 (1990).
8. J.M.G. Levins, D.M. Benton, J. Billowes *et al.*, Phys. Rev. Lett. **82**, 2476 (1999).
9. H. De Vries, C.W. De Jager, C. De Vries, At. Data Nucl. Data Tables **36**, 495 (1987).
10. I. Tanihata, Prog. Part. Nucl. Phys. **35**, 505 (1995).
11. P.G. Hansen, A.S. Jensen, B. Jonson, Annu. Rev. Nucl. Part. Sci. **45**, 591 (1995).
12. J.Y. Zeng, Acta Phys. Sin. **13**, 357 (1957); **24**, 151 (1975).
13. B. Nerlo-Pomorska, K. Pomorski, Z. Phys. A **344**, 359 (1993).
14. Y.K. Gambhir, S.H. Patil, Z. Phys. A **321**, 161 (1985); **324**, 9 (1986).
15. C.Y. Tseng (J.Y. Zeng), T.S. Cheng, F.C. Yang (F.J. Yang), Nucl. Phys. A **334**, 470 (1980).
16. J.Y. Zeng, F.C. Yang, Sci. Sin. (Ser. A) **25**, 263 (1982).
17. P. Mueller, J.R. Nix, W.D. Myers., W.J. Swiatecki, At. Data Nucl. Data Tables **59**, 185 (1995).
18. B. Nerlo-Pomorska, K. Pomorski, Z. Phys. A **348**, 168 (1994).
19. J. Dobaczewski, W. Nazarewicz, T.R. Werner, Z. Phys. A **354**, 27 (1996).
20. M. Warda, B. Nerlo-Pomorska, K. Pomorski, Nucl. Phys. A **635**, 484 (1998) and references therein.
21. B.D. Serot, J.D. Walecka, Adv. Nucl. Phys. **16**, 1 (1986).
22. P-G. Reinhard, Rep. Prog. Phys. **52**, 439 (1989).
23. P. Ring, Prog. Part. Nucl. Phys. **37**, 193 (1996).
24. J. Meng, Nucl. Phys. A **635**, 3 (1998); **654**, 702c (1999).
25. J. Meng, K. Sugawara-Tanabe, S. Yamaji, P. Ring, A. Arima, Phys. Rev. C **58**, R628 (1998).
26. J. Meng, K. Sugawara-Tanabe, S. Yamaji, A. Arima, Phys. Rev. C **59**, 154 (1999).
27. J. Meng, P. Ring, Phys. Rev. Lett. **77**, 3963 (1996).
28. J. Meng, P. Ring, Phys. Rev. Lett. **80**, 460 (1998).
29. J. Meng, I. Tanihata, S. Yamaji, Phys. Lett. B **419**, 1 (1998).
30. J. Meng, S.-G. Zhou, I. Tanihata, to be published in Phys. Lett. B.
31. M.M. Sharma, M.A. Nagarajan, P. Ring, Phys. Lett. B **312**, 377 (1993).
32. G. Audi, A.H. Wapstra, Nucl. Phys. A **595**, 409 (1995).
33. E.P. Wigner, *Proceeding of the Robert A. Welch Foundation Conference on Chemical Research*, edited by W.D. Millikan, Vol. **1** (Robert A. Welch Foundation, Houston TX, 1957) p. 167.